some methods are mentioned for estimating the slopes by various means (e.g., Akima's method). There are also some hints on finding monotone interpolating cubic splines, in case the data are monotone, by imposing certain conditions on the slopes. Some error estimates are quoted from de Boor (1). A method for finding cubic splines only from values at the breakpoints is postponed to Chapter 7.

In Chapter 7 on *Cubic Polynomial Space Curve Splines* there is a cubic interpolation formula for connecting two points in $\mathbf{R}^3$ with prescribed slopes (also in $\mathbf{R}^3$). The question of how to find such a formula is, however, not raised. Most of the material of this section is hidden in exercises. At the end Bernstein–Bézier curves are mentioned and the fact that they do not interpolate the control points (apart from the first and the last one) is emphasized.

Chapter 9 on *Global Cubic Space Curve Splines* is, apart from the beginning chapter and Chapter 15 with two C programs, the longest. It contains the usual derivations of cubic splines (interpolating at the breakpoints) as solution of tridiagonal linear systems. Minimal properties are also given. It is not quite clear why at this point long derivations occur where at other places just results are given.

We could continue refereeing all further chapters and some details might be of interest. However, there is some doubt in general whether "the discursive style employed" is of much use for a greater audience. There are several figures in the book, all of them untitled. This makes it occasionally difficult to find the corresponding connection with the text. There are no worked numerical examples at all. The references (three pages) range in time from 1946 to 1995. Most (but not all) of the quoted authors are listed by first and last name. In general, exercises are welcome in a book, but here, where the many exercises are distributed over the whole text (sometimes including solutions), they disturb the fluent reading of the material. The idea of always insisting on interpolation at prescribed points is questionable. Good approximations to curves (or surfaces) generally also interpolate at some points which are not known in advance.

To answer the question from the beginning of this review: it might be possible to learn something from this book, but without any joy. A mathematically trained person should better consult other books.

REFERENCES

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Gheorghe Micula and Sanda Micula, *Handbook of Splines*, Mathematics and its Applications **462**, Kluwer, Dordrecht, 1999, xvi + 604 pp.

Splines play a fundamental role in applied mathematics and in various fields of applications. In the last 50 years, many thousands of papers on splines were published. The reason is that splines are efficient tools for solving a variety of problems because they possess nice structural properties and have excellent approximation powers and fast algorithms for computing splines are available (although the important multivariate problems are not solved completely). The aim of this book is to give an introduction to the theory of splines together with applications to various fields such as data fitting, interpolation, approximation, computer aided design, integral equations, differential equations, stochastics, and wavelets.

Chapter 1 contains well-known results on B-spline bases and on interpolation by natural, discrete, and periodic splines. Moreover, the relation of splines and the optimal approximation of linear functionals is investigated. In Chapter 2 the Bézier representation, the dimension and the existence of local bases functions for bivariate spline spaces are discussed. Moreover some aspects of the approximation power and of interpolation by such spaces are given. Chapter 3 deals with nonlinear sets of splines (rational and regular splines) and their applications in solving ordinary differential equations. Chapter 4 deals with the numerical treatment of integral equations by using splines. The spline Galerkin method and the spline collocation method for solving Fredholm and Volterra integral equations and integro-differential equations are described. Chapter 5 describes the numerical solution of ordinary boundary value problems as well as delay differential equations with the aid of splines. The connection of collocation methods and methods of Runge–Kutta type are studied. In Chapter 6, some results on Lagrange and Hermite interpolation by finite elements functions are given. Chapter 7 is devoted to the solution of boundary value problems for partial differential equations. Finite element methods for solving elliptic Dirichlet and Neumann problems and spline collocation methods for solving parabolic and hyperbolic problems are studied. Chapter 8 describes the use of spline curves and tensor product surfaces in computer aided geometric design, in particular the approach of non-uniform rational B-splines. In Chapter 9 a shape model, which combines deterministic splines and stochastic fractals, is discussed. Variational splines in one variable are introduced and how to discretize the spline energy expressions using finite elements is described. In Chapter 10 properties of box splines and multivariate truncated power functions, in particular recurrence relations and the behavior of box spline series, are presented. Chapter 11 gives a brief introduction to spline wavelets, in particular to the use of univariate cardinal B-splines to generate a multiresolution analysis.

The book is addressed to researchers and scientists interested in applications. It gives an informative introduction to splines and to several fields of applications in which splines play a fundamental role. For a more detailed study of the topics, the reader is referred to the extensive list of about 6000 papers in the references.

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Sergei Yu. Slavyanov and Wolfgang Lay: *Special Functions, A Unified Theory Based on Singularities*, Oxford Mathematical Monographs, Oxford University Press, 2000, xvi + 293 pp.

The central issue of this monograph is Heun's differential equation. This equation generalizes the hypergeometric equation by the addition of one more finite regular singularity. The equation itself never really penetrated into the domain of physical sciences, except for the case of Lamé's equation, where the singularities are specialized to be elementary. The additional singularity, however, creates the possibility of several (and new) types of confluences, and exactly the confluent versions of the equation show up in many different fields (e.g., the spheroidal wave equation, Mathieu equation, several Schrödinger cases), often giving rise to new types of eigenvalue problems. The class of Heun equations is presented here in a unifying, singularity-based, framework. On one side, the class is extending the hypergeometric class in the above-mentioned sense, while in another direction, the equations are shown to be in one-to-one correspondence with the nonlinear equations of the Painlevé class.

Chapter 1 introduces the relevant tools from the general theory of linear second-order ODEs with polynomial coefficients: s-rank classification and principles of confluence, Frobenius versus Thomé expansions, the generalized Riemann scheme, s-homotopic, Möbius, and Jaffé transformations, the central two-point connection problem, and the Birkhoff set of asymptotic solutions to the Poincaré–Perron difference equation for the expansion coefficients. In Chapter 2 these tools are applied to reconsidering the hypergeometric class in the unifying framework. The chapter includes an original treatment of the difference equations satisfied by hypergeometric and confluent hypergeometric functions and a section about the related classical orthogonal polynomials. Following the same lines, the Heun class of equations